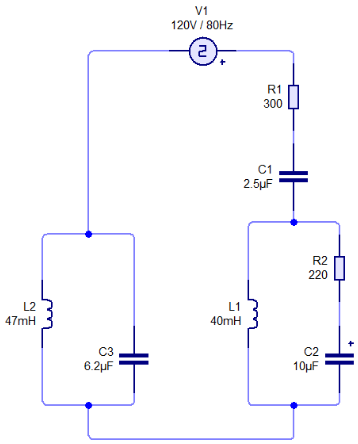
Example AC Circuit Analysis

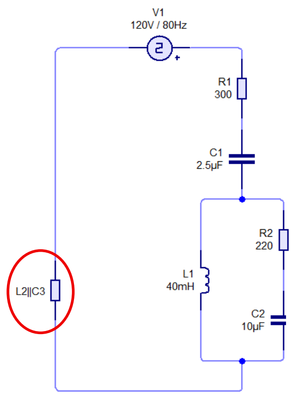


a. Find the current and voltage value through each component in the circuit

Step 1: Work out Reactance

- $X_{R1} = 300$
- $X_{R2} = 220$
- $X_{L1} = 2\pi f L = 2\pi * 80 * 0.04 = 20.1062$
- $X_{L2} = 2\pi f L = 2\pi * 80 * 0.047 = 23.6248$
- $X_{C1} = \frac{1}{2\pi f C} = \frac{1}{2\pi * 80 * 0.0000025} = 795.7747$
- $X_{C2} = \frac{1}{2\pi f C} = \frac{1}{2\pi * 80 * 0.000010} = 198.9437$
- $X_{C3} = \frac{1}{2\pi f C} = \frac{1}{2\pi * 80 * 0.0000062} = 320.8769$

Step 2: Combine C3 and L2 in parallel making an equivalent component (L2||C3)



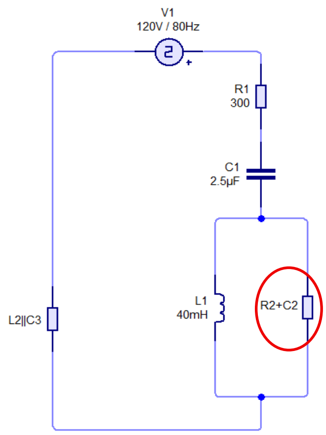
$$\boldsymbol{\frac{1}{\frac{1}{X_R} + \frac{1}{X_L - X_C} i}}$$

$$\frac{1}{\frac{1}{23.6248i} - \frac{1}{320.8769i}} = \frac{1}{0.04232i - 0.003116i} = \frac{1}{0.3920i} = \mathbf{25.5067\Omega\angle 90°}$$

Then convert back to cartesian to make it easier to work with

$(0 + 25.5067i)$

Step 3: Combine R2 and C2 in series making an equivalent component (R2+C2)



$(220 - 198.9437i)$

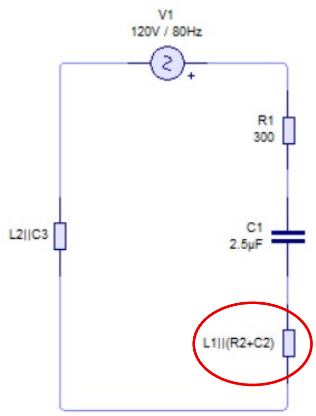
Turn into polar

$$|Z_{tot}| = \sqrt{R^2 + X^2} = \sqrt{220^2 + (-198.9437i)^2} = 296.6119$$

$$\theta = \tan^{-1}\left(\frac{X}{R}\right) = \tan^{-1}\left(\frac{-198.9437i}{220}\right) = -42.1227°$$

$$\mathbf{296.6119\Omega\angle -42.1227°}$$

Step 4: Combine R2 + C2 and L1 in parallel making an equivalent component (L1||(R2+C2))



$$\frac{1}{\frac{1}{(220-198.9437i)}+\frac{1}{(0+20.1062i)}}$$

First we work out the admittance of the (R2+C2)

$$\frac{1}{(a+bi)}=\frac{(\boldsymbol{a-bi})}{\boldsymbol{a^2+b^2}}=\frac{220+198.9437i}{220^2+198.9437^2}$$

$$\frac{220+198.9437i}{87978.5958}=0.002501+0.002261i$$

Note that the top of the second fraction (highlighted in bold) is $-bi$ flipping the sign of the imaginary component

Next we work out the admittance of L1

$$\frac{1}{Z_{eq}}=-\frac{1}{20.1062}=-0.04974i$$

Then we add the two admittances

$$Y_{eq}=(0.002501+0.002261i)+(0-0.04974i)=(0.002501-0.04748i)$$

Then we convert back to impedance

$$Z_{eq}=\frac{1}{(0.002501-0.04748i)}$$

$$\frac{1}{(a+bi)}=\frac{(a-bi)}{a^2+b^2}=\frac{(0.002501+0.04748i)}{0.002261}=(1.1061+20.9996i)$$

Finally lets convert to polar

$$|Z_{tot}|=\sqrt{R^2+X^2}=\sqrt{1.1061^2+20.9996i^2}=21.0287$$

$$\theta=\tan^{-1}\left(\frac{X}{R}\right)=\tan^{-1}\left(\frac{20.9996}{1.1061}\right)=86.9849$$

$$\mathbf{21.0287\Omega\angle 86.9849°}$$

Step 5: Combine all components in series making an equivalent component

	Resistance	Reactance
R1	300	0
C1	0	-795.7747
L1||(R2+C2)	1.1061	20.9996
L2||C3	0	25.5067
Total	**301.1061**	**-749.2684*i***

Then convert it into polar

$$|Z_{tot}| = \sqrt{R^2 + X^2} = \sqrt{301.1061^2 + (-749.2684i)^2} = 807.5073$$

$$\theta = \tan^{-1}\left(\frac{X}{R}\right) = \tan^{-1}\left(\frac{-749.2684i}{301.1061}\right) = -68.1064°$$

$$\mathbf{807.5073\Omega\angle - 68.1064°}$$

Step 6: Work out the source current from the AC source

$$V = IZ \rightarrow I = \frac{V}{Z} = \frac{120}{807.5073} = 0.1486 = \mathbf{148mA}$$

Step 7: Based on current and impedance values work out the voltage drop through the main series components

Value	R1	C1	L1||(R2+C2)	L2||C3
V				
I	$0.1486A$	$0.1486A$	$0.1486A$	$0.1486A$
Z	$300\angle 0°$	$795.7747\angle - 90°$	$21.0287\angle 86.9849°$	$25.5067\Omega\angle 90°$

R1: $V = IZ = 0.1486 * 300 = \mathbf{44.58}$

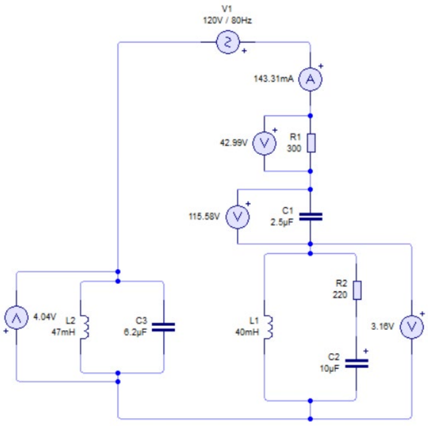
C1: $V = IZ = 0.1486 * 795.7747 = \mathbf{118.2521}$

L1||(R2+C2): $V = IZ = 0.1486 * 21.0287 = \mathbf{3.1249}$

L2||C3: $V = IZ = 0.1486 * 25.5067 = \mathbf{3.7903}$

Value	R1	C1	L1||(R2+C2)	L2||C3
V	44.58	118.2521	3.1249	3.7903
I	$0.1486A$	$0.1486A$	$0.1486A$	$0.1486A$
Z	$300\angle 0°$	$795.7747\angle - 90°$	$21.0287\angle 86.9849°$	$25.5067\Omega\angle 90°$

Lets check our values against the simulation



Though we're off by a small amount its most likely due to rounding withing circuit wizard we are within 5%.

Step 8: Work out the current through each branch of L1||(R2+C2)

Using the current divider equation for each branch

$$I_{L1} = \frac{Z_{R2+C2}}{Z_{L1}+Z_{R2+C2}} * I_{tot}$$

$$I_{R2+C2} = \frac{Z_{L1}}{Z_{L1}+Z_{R2+C2}} * I_{tot}$$

We need to work out the denominator using complex addition

$$Z_{L1} + Z_{R2+C2} = (0 + 20.1062i\ ) + (220 - 198.9437i) = (220 - 178.8375i)$$

Then convert this into polar

$$|Z_{tot}| = \sqrt{R^2 + X^2} = \sqrt{220^2 + (-178.8375i)^2} = 283.5187$$

$$\theta = \tan^{-1}\left(\frac{X}{R}\right) = \tan^{-1}\left(\frac{-178.8375i}{220}\right) = -39.1076°$$

$$283.5187\Omega\angle - 39.1076°$$

Then just apply this value and values we already have into the equation

$$I_{L1} = \frac{296.6119}{283.5187} * 0.1486 = 0.1555 = \mathbf{155}\boldsymbol{mA}$$

$$I_{R2+C2} = \frac{20.1062}{283.5187} * 0.1486 = 0.01054 = \mathbf{10.54}\boldsymbol{mA}$$

Step 9: Work out the voltage through all components of L1||(R2+C2)

$$V_{L1} = IZ = 20.1062 * 0.1555 = 3.1265$$

$$V_{R2} = 220 * 0.01054 = 2.3188$$

$$V_{C2} = 198.9437 * 0.01054 = 2.0968$$

Step 10: Work out the current through each branch of L2||C3

Using the current divider equation for each branch

$I_{L2} = \frac{Z_{C3}}{Z_{L2}+Z_{C3}} * I_{tot}$

$I_{C3} = \frac{Z_{L2}}{Z_{L2}+Z_{C3}} * I_{tot}$

We need to work out the denominator using complex addition

$Z_{L2} + Z_{C3} = (0 + 23.6248i\ ) + (0 - 320.8769i) = (0 - 297.2521)$

Then convert into polar

$297.2521\angle - 90$

Then just apply this value and values we already have into the equation

$I_{L2} = \frac{320.8769}{297.2521} * 0.1486 = 0.1604A = 160.4mA$

$I_{C3} = \frac{23.6248}{297.2521} * 0.1486 = 0.0118A = 11.8mA$

Step 11: Work out the voltage through all components of L2||C3

$V_{L2} = IZ = 23.6248 * 0.1604 = 3.7894$

$V_{C3} = 320.8769 * 0.0118 = 3.7863$